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A MODEL OF BUBBLES RISING IN A FLUIDIZED BED

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Abstract—A model for a single fully developed bubble moving in an unbounded fluidized bed is presented. The model allows bubble growth or shrinkage during the rise inside the bed, as well as dependence of the rise velocity upon specified bed parameters. Limiting cases of nearly spherical bubbles and of sufficiently large bubbles whose form resembles that of a spherical segment are considered in more detail. The form of bubbles rising in either fluidized beds or one-phase liquids, and its dependence on the effective "surface tension" acting on the bubble boundary are discussed.

1. INTRODUCTION

Real particulate beds fluidized with a gas or even with a dense liquid are usually non-uniform in the sense that spontaneous formation of voids filled mainly with the fluidizing fluid and containing no particles takes place (e.g. Rowe & Partridge 1965; Rowe 1971). Sometimes such voids or "bubbles" are provoked artificially by injecting a pulse or a jet of the fluid into a bed. In both cases the behaviour of resulting bubbles has much in common with that of a large bubble rising through an extended liquid studied firstly by Davies & Taylor (1950). While travelling in a bed those bubbles favour intensification of the mixing of both the dispersed solid phase (Rowe, Partridge, Cheney, Henwood & Lyall 1965; Potter 1971) and the continuous fluid phase (Davies & Richardson 1966; Partridge & Rowe 1966) and influence, thereby, heat and mass exchange processes in the bed. This stimulated rather intensive and systematic study of fluidization bubbles during the last decade.

The first model of a bubble in an extended uniform fluidized bed was suggested by Davidson (1961) who considered simultaneously the irrotational flow of a continuum representing the particulate phase around the bubble, and the filtration of the interstitial fluid within a porous body formed by moving particles (see also Davidson & Harrison 1963). Jackson (1963) accepted a hypothesis that there is a strict analogy between large bubbles in fluidized beds and in liquids and endeavoured to analyze additionally the bed structure in the vicinity of the bubble upper surface. It has to be noted in this connection that there are no direct unequivocal foundations for such an assumption so that his results have to be regarded as tentative and purely suggestive in nature, despite a certain success in confirmation of some his conclusions by experiments in Lockett & Harrison (1967). Another model of the steady motion of a single bubble was brought into existence by Murray (1965) who used a modified Oseen technique when considering the flow around two- or three-dimensional bubbles.

All these models were carefully examined and compared with experimental evidence and among themselves. They were also developed in such a way as to include into analysis non-stationary effects accompanying initial motion of bubbles (Murray 1967; Collins 1971), the interaction with other bubbles and with walls of a container (Collins 1969; Clift, Grace, Cheung & Do 1972), pecularities of the fluid flow through both the bubble and the adjoining cloud with closed circulation patterns (Hargreaves & Pyle 1972), etc.

Comparison with experimental data showed these theories to be sufficiently adequate in the matter of correct qualitative description of many fundamental bubble properties. There are, nonetheless, systematic deviations of theoretical conclusions from observed phenomena. Thus, the permanent growth or contraction of rising bubbles has not hitherto been explained; there is no sound explanation for the dependence of the rise velocity upon bed parameters such as the fluidization velocity and the bed voidage (porosity); it is unknown so far what possible reasons are responsible for the bubble form, and so on. Moreover, there exists an inherent defect characteristic of the theories mentioned, which lies in presuming the bubble form to be given *a priori*, whereas it ought to be found from the solution of the corresponding mathematical problem.

In what follows, an attempt is made to revise the formulation of the boundary problem for a fully developed fluidization bubble and to construct a new physical model which would account for some additional factors affecting its behaviour. In order to make the consideration as simple and comprehensible as possible and to leave the key ideas unencumbered with laborous calculations and other non-essentials, attention is primarily focused on the principal aspects of the model, and certain simplifying assumptions are brought into action. Various conventional problems which may be looked upon as irrelevant to the predominant aim of the model formulation are also disregarded, whether such problems are of practical significance or not. For example, attention is paid neither to solids nor to fluid flow patterns around and within the bubble under study, although the corresponding streamlines could be readily computed by standard methods.

2. FORMULATION OF THE PROBLEM

To describe the flow of both particulate and fluid phases outside a single bubble moving in an otherwise uniform extended fluidized bed, one must use equations of mass and momentum conservation for the phases of a concentrated suspension. A set of such equations was rigorously derived in Buyevich & Markov (1973) under a condition of weak pulsating motion of the fluid and particles. In the Eulerian approximation when viscous stresses are neglected those equations can be written as follows:

$$\operatorname{div}(\epsilon \mathbf{v}) = 0, \qquad \operatorname{div}(\rho \mathbf{w}) = 0, \tag{2.1}$$

$$d_0\epsilon(\partial/\partial t + \mathbf{v}\nabla)\mathbf{v} = -\nabla p + d_0\mathbf{g} - \mathbf{f}(\rho, \mathbf{u}), \tag{2.2}$$

$$d_1\rho(\partial/\partial t + \mathbf{w}\nabla)\mathbf{w} = \rho(d_1 - d_0)\mathbf{g} + \mathbf{f}(\rho, \mathbf{u}), \tag{2.3}$$

where p is the mean fluid pressure, $\mathbf{v}$ and $\mathbf{w}$ are the mean velocities of the fluid and particulate phases, respectively, ϵ and $\rho = 1 - \epsilon$ are the bed voidage and the volume concentration of particles, d_0 and d_1 are the densities of the fluid and particle material, $\mathbf{g}$ is the gravity acceleration vector and $\mathbf{f}(\rho, \mathbf{u})$ is the interphase interaction force per unit volume of the mixture depending on the relative interstitial fluid velocity $\mathbf{u} = \mathbf{v} - \mathbf{w}$ (a superficial velocity commonly used equals $\epsilon\mathbf{u}$). When dealing with [2.1]–[2.3], the phases are considered as co-existing interpenetrating continua. If $d_0 \ll d_1$ these equations turn into those used earlier by Jackson (1963) and Murray (1965).

Equations [2.1]–[2.3] are somewhat insufficient because random pulsations of the fluid and particles ("pseudo-turbulence"), which have been shown by Buyevich (1971, 1972b) to exert an essential influence on rheological properties of a fluid–solid mixture, are completely unaccounted for. An analysis based on the evaluation of various pseudo-turbulent characteristics in Buyevich (1972b) testifies that the most important consequence of pulsation consists in the appearance of additional stresses affecting the average flow of both phases. The effect of the pseudo-turbulence on "quasi-viscous" stresses is of no interest here because of utilization in the treatment of the Eulerian approximation. Thus, only extra normal stresses have to be introduced in addition into [2.1]–[2.3]. Note that experimental measurement of normal stresses in a fluidized bed was reported, for instance, in Meissner & Kusik (1970).

The equilibrium normal stress tensors $\mathbf{P}_{0\infty}$ and $\mathbf{P}_{1\infty}$ prescribed to the fluid and particulate phases, respectively, are realized in a uniform steady flow when ϵ, $\mathbf{v}$ and $\mathbf{w}$ do not depend on time and coordinates. They represent single-valued tensor functions of ϵ (or ρ) and $\mathbf{u}$ which were calculated for a special case of a suspension with comparatively small particles in Buyevich (1972b). Real flows, particularly those around moving bubbles, are usually unsteady and non-uniform. To find the normal stress tensors $\mathbf{P}_0$ and $\mathbf{P}_1$ for such a "non-equilibrium" state, one must solve the kinetic equation for suspended particles derived by Buyevich (1972a). Thus, there are in a general case eight scalar equations for eight unknown functions ϵ, p, v_i and w_i. These equations involve the tensors $\mathbf{P}_0$ and $\mathbf{P}_1$ whose dependence on the above functions is to be found after solution of an additional (kinetic) equation, so that a rather complicated mathematical problem arises.

Other stresses occur in the particulate phase due to direct contacts between adjoining particles. These stresses, whose origin is in principle the same as that of stresses in granular solids, are negligible for a system of freely suspended particles, so that they vanish in the equilibrium state far away from a bubble. However, the situation may well be different in the bubble vicinity where permanent particle contacts are eventually possible due to changes in the fluid flow distribution accompanying preferable fluid transfer through the bubble, rather than through surrounding volumes of the bed. In other words, the bubble exerts the "draining" influence upon the bed forcing the particles to be brought closer together, and the additional stresses to appear.

It is hardly possible nowadays to derive an exact and rigorous solution of such a problem without further simplifying suggestions. A natural way of overcoming this difficulty might consist in constructing a suitable system of successive approximations and consecutive investigation of the corresponding results. For the main purposes pursued in this paper it is quite sufficient to treat only the lowest approximation which has to be chosen, of course, so as not to violate the physical meaning of the problem.

As a first approximation we consider the "incompressible" flow of the particulate phase occurring when the bed voidage is uniform. Acceptance of this assumption meets with two serious complications. First, this reduces the number of unknown variables involved in the equations so that one of the latter appears to be superfluous and, second, the kinetic equation becomes then rather meaningless because of damaging the basic philosophy laid in its foundation. The first difficulty was in fact encountered by Murray (1965), for the assumption of incompressibility proved to be consistent only with the linearized set of equations in his paper. Obviously, one should account above all for the fact that the Murray's original equations do not permit this assumption to be made and regard its consistency with the linearized set as an accidental piece of luck rather than an intrinsic feature of the linearization procedure. It is therefore evident that a thorough consideration is sorely needed before assuming ϵ to be constant.

The first difficulty can be resolved if one takes into consideration that the assumption of incompressibility of the particulate phase flow requires the effective pressure of this phase to be regarded as an independent variable. This is an irrevocable condition of self-consistency of such an assumption, and the situation resembles in this respect that encountered in the conventional hydrodynamics of one-phase media. The requirement of constant porosity can be perforce imposed upon the system under study provided that a reason leading to its actual satisfaction is simultaneously brought into action. Physically, it is quite clear that the only possible reason opposing particles which tend to group together or to form a loosened structure, consists in the interparticle interaction being effected either by way of their direct contacts or through the fluid pressure and velocity fields in the interstitial space. The simplest and, perhaps, most sensible manner to account for this interaction is to introduce into the equations the effective particulate phase pressure, as was done previously by Davidson & Harrison (1963). Note in this connection that Murray (1965) not only could but also should involve such a pressure in his analysis.

The normal stresses resulting from particle pulsations certainly contribute to this pressure. However, the latter is already considered as an unknown variable subject to determination while solving the governing equations. Therefore, there is no need to look for these stresses in an explicit form and, consequently, to bother with the kinetic equation. It is sufficient to include formally the relevant longitudinal (i.e. directed along $\mathbf{u}$) component of these stresses into this variable. Analogously, the longitudinal fluid normal stress caused by pulsations can be added to the unknown fluid pressure p. The corresponding lateral components can be neglected altogether because they are small compared with the longitudinal ones (Buyevich 1972b). Thus, the second complication above-mentioned is effectively avoided.

Several assumptions are further made in order to simplify the calculation. First of all, a linear relationship

$$\mathbf{f}(\rho, \mathbf{u}) = \rho\beta K\mathbf{u}, \qquad K = K(\rho) \tag{2.4}$$

is supposed to be valid, β being a constant depending on physical parameters of the phases and $K(\rho)$ equalling unity for a dilute system ($\rho \to 0$) and monotonously increasing as ρ grows. Both β and K are regarded as known quantities following from either theory or experiment. Strictly speaking, [2.4] is true for beds of very fine particles when the Reynolds number is smaller than unity. It was nevertheless applied, because of its simplicity, even to beds of fairly large particles whose Reynolds number markedly exceeds unity in all papers on the topic of which the author is aware. In that case [2.4] has to be looked upon as a convenient approximate formula.

We suppose the flow to be irrotational everywhere outside the bubble under consideration and a possible wake region behind it. And, last, we imagine the flow around the bubble to be almost stationary and neglect time derivatives in [2.2] and [2.3]. The latter assumption is easily conceivable if the bubble preserves its own volume as happens with bubbles in a liquid when phase transitions are absent. However, this is not the case in the present situation since there are no sound reasons preventing a fluidization bubble from changing its volume. The stationarity condition is approximately true if the rate of volume changing is sufficiently small. This means the velocity of the bubble surface relative to its gravity centre is well below the rise velocity.

By accounting for all these assumptions and introducing into [2.2] and [2.3] the normal stresses one is able to rewrite those equations in the following form:

$$\nabla(\tfrac{1}{2}d_0\epsilon v^2) = -\nabla(p + P_0 - d_0\mathbf{gr}) - \rho\beta K\mathbf{u} \tag{2.5}$$

$$\nabla(\tfrac{1}{2}d_1\rho w^2) = -\nabla(P_1 - \rho(d_1 - d_0)\mathbf{gr}) + \rho\beta K\mathbf{u}. \tag{2.6}$$

Here P_0 and P_1 denote the longitudinal normal stresses in the fluid and particulate phases, respectively, and [2.4] is used. These equations together with [2.1] serve for determination of eight functions $p + P_0$, P_1, v_i and w_i. Note that the lateral normal stresses due to the pseudo-turbulence fall out the analysis in accordance with the remark above.

Equations of the same kind as [2.5] and [2.6] were actually used previously in Davidson & Harrison (1963). There, a moment contribution due to local momentum exchange processes which may be loosely termed as an "interparticle" interaction was intuitively included into the analysis in a similar manner. In this respect equations in Davidson & Harrison (1963) are physically more correct than those in Jackson (1963) and Murray (1965) because the latter do not involve the particulare phase "pressure" P_1 and a difficulty consisting in a lack of one unknown variable arises on condition of constant voidage.

It is advisable to introduce potentials ϕ and ψ for the particulate phase flow and for the fluid filtration within interstices, respectively,

$$\mathbf{w} = \nabla\phi, \qquad \mathbf{v} = \nabla(\phi + \psi). \tag{2.7}$$

Then [2.1], [2.5] and [2.6] yield

$$\Delta\phi = 0, \qquad \Delta\psi = 0, \tag{2.8}$$

$$\rho\beta K\psi = -\Pi_0 = \Pi_1, \tag{2.9}$$

where effective dynamic pressures Π_0 and Π_1 are defined as follows:

$$\Pi_0 = p + P_0 + \tfrac{1}{2}d_0\epsilon v^2 - d_0\mathbf{gr}, \tag{2.10}$$

$$\Pi_1 = P_1 + \tfrac{1}{2}d_1\rho w^2 - \rho(d_1 - d_0)\mathbf{gr}. \tag{2.11}$$

Equations [2.7]–[2.11] give an opportunity to investigate all the quantities of interest. Obviously, to describe the fluid flow inside the bubble under consideration one must use the conventional Eulerian equations for a homogeneous one-phase medium.

We consider a bubble whose surface is determined in a given moment by a relation

$$r = r_0(\theta) = a[1 + \xi(\theta)], \quad \xi(\pi) = 0, \tag{2.12}$$

written in a co-ordinate system connected with the bubble (see figure 1). In accordance with the above assumption that the flow differs from the stationary one only slightly, one may neglect the dependence of parameters in [2.12] upon time while solving the hydrodynamic problem.

In the laboratory system of co-ordinates the velocity w is presumed to be zero far from the bubble so that one has in the "convective" co-ordinate system used

$$\nabla(\phi + \psi) = \mathbf{v}_\infty = \mathbf{u}_\infty + \mathbf{w}_\infty, \qquad \nabla\phi = \mathbf{w}_\infty, \quad r \to \infty, \tag{2.13}$$

where $\mathbf{w}_\infty$ is a vector to be determined afterwards. Similarly,

$$p + P_0 = p_\infty + P_{0\infty}, \qquad P_1 = P_{1\infty}, \qquad r \to \infty. \tag{2.14}$$

Here p_∞ and $\mathbf{u}_\infty$ are the pressure and the interstitial velocity far away from the bubble given by obvious relations following from [2.5] and [2.6],

$$p_\infty = (d_0\epsilon + d_1\rho)\mathbf{gr}, \qquad \mathbf{u}_\infty = -(\beta K)^{-1}(d_1 - d_0)\mathbf{g}. \tag{2.15}$$

The equilibrium functions $P_{0\infty}$ and $P_{1\infty}$ can be represented on the basis of the results in Buyevich (1972b) as

$$P_{0\infty} = \tfrac{1}{2} d_0 \epsilon L_0 u^2, \qquad P_{1\infty} = \tfrac{1}{2} d_1 \rho L_1 u^2, \qquad r \to \infty, \tag{2.16}$$

where L_0 and L_1 are functions of ϵ, their order of magnitude being unity for values of ϵ usually encountered in fluidized beds. It follows from [2.10] and [2.11] and [2.14]–[2.16]

$$\Pi_0 = \Pi_{0\infty} = \tfrac{1}{2} d_0 \epsilon (v_\infty^2 + L_0 u_\infty^2) + \rho(d_1 - d_0)\mathbf{gr}, \qquad r \to \infty, \tag{2.17}$$

$$\Pi_1 = \Pi_{1\infty} = \tfrac{1}{2} d_1 \rho (w_\infty^2 + L_1 u_\infty^2) - \rho(d_1 - d_0)\mathbf{gr}, \qquad r \to \infty. \tag{2.18}$$

Equations [2.13] and [2.17], [2.18] give boundary conditions which have to be satisfied at large distances from the bubble.

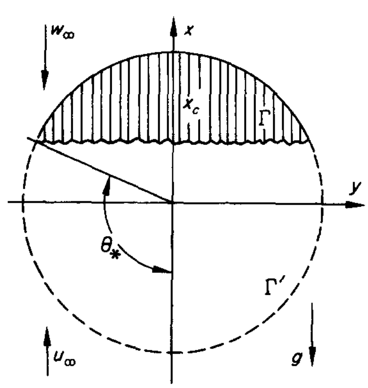


Figure 1. Schematic representation of the bubble and wake region.

Now we proceed to formulation of boundary conditions at the surface Γ' enclosing both the bubble and the adjoining wake region and at the bubble surface Γ (figure 1). Clearly, the normal component of the velocity of the particulate phase at Γ must coincide with that of the surface Γ itself. Supposing that the bubble is growing or shrinking without changes in its form we get

$$\frac{\partial \phi}{\partial n} = w_0 \frac{r_0(\theta)}{r_0(\pi)} = w_0[1 + \xi(\theta)], \quad \mathbf{r} \in \Gamma. \tag{2.19}$$

Here w_0 is an unknown quantity representing the velocity of the critical frontal point $r = a$, $\theta = \pi$ on the bubble surface. This quantity can be found from a condition of volume conservation written in the form

$$\oint \left[\epsilon \frac{\partial(\phi + \psi)}{\partial n} + \rho \frac{\partial \phi}{\partial n}\right] \mathrm{d}\Gamma = \oint \left(\epsilon \frac{\partial \psi}{\partial n} + \frac{\partial \phi}{\partial n}\right) \mathrm{d}\Gamma = 0. \tag{2.20}$$

This condition is representative of the fact that there are no sources or sinks inside the bubble. Note that integration over Γ in [2.20] may be substituted by integration over any closed surface surrounding the bubble.

A necessary physical condition enabling a sharp boundary between the bubble and the ambient fluidized bed to exist requires the effective "pressure" of a continuum, modelling the particulate phase, to vanish at the boundary; that is P_1 must equal zero. Keeping in mind that

$$\Pi_0 + \Pi_1 = \Pi_{0\infty} + \Pi_{1\infty} = \text{const} \tag{2.21}$$

follows from [2.9] and calculating the constant in [2.21] with the help of [2.17] and [2.18], one obtains after accounting for [2.9] and [2.11]

$$P_1 = \tfrac{1}{2} d_0 \epsilon (v_\infty^2 + L_0 u_\infty^2) + \tfrac{1}{2} d_1 \rho (w_\infty^2 - |\nabla \phi|^2 + L_1 u_\infty^2) + \rho(d_1 - d_0)\mathbf{g}\mathbf{r} + \rho\beta K \psi = 0, \qquad \mathbf{r} \in \Gamma \tag{2.22}$$

Conditions of continuity of the fluid flux and normal stress at the bubble surface must be satisfied, the motion of the surface itself being taken into account. Considering that the treatment of the flow inside the bubble is irrelevant to the intended purpose of the paper, one is free to disregard the former condition. The latter takes the form

$$\Pi_0 \sim \psi = C, \qquad \mathbf{r} \in \Gamma \tag{2.23}$$

where C is a constant. This condition results from the fact that the resistance to fluid filtration through a porous body formed by particles is much greater than that to the flow within the bubble where particles are practically absent. Therefore, the dynamic fluid pressure is approximately constant throughout the bubble as compared with a similar quantity outside the bubble. Equation [2.23] was confirmed by experiments in Reuter (1963).

When there is no wake behind the bubble, the boundary problem for [2.8] with conditions [2.13], [2.19], [2.20], [2.22] and [2.23] is consistent. It is easy to see that in this case there is a superfluous boundary condition at Γ which must be satisfied at the expense of the corresponding choice of the form of Γ, i.e. of [2.12], and determines as well the constants involved. The situation is more complicated when Γ' does not coincide with Γ; that is there exists a wake where the flow of both phases is undoubtedly rotational, and the equation for ϕ in [2.8] is not defined everywhere within the bed. Thus the above formulation is clearly deficient in that the motion inside the wake falls out of the analysis altogether. However, as will be seen later, it becomes possible to draw important conclusions even in this case.

3. BUBBLES WITH NO WAKE

We begin with consideration of a nearly spherical bubble assuming the wake region to be absent or negligible. Such a bubble can be regarded as an idealized preliminary model for real bubbles occurring in practice, especially for comparatively small bubbles rising through a uniformly fluidized bed of fine particles. All the previous analyses of three-dimensional bubbles were essentially restricted to study of spherical bubbles.

The function $\xi(\theta) \ll 1$ in [2.12] can now be used as a small parameter for formulation of a calculation scheme. As a first approximation, fairly sufficient for the present purposes, we take the flow near a sphere with the radius a, $\xi(\theta)$ being identically equal to zero. Solving [2.8] with conditions [2.13], [2.19] and [2.23] yields

$$\phi = w_\infty\left(1+\frac{a^3}{2r^3}\right) r \cos\theta - w_0\frac{a^2}{r} \quad , \qquad [3.1]$$

$$\psi = -\frac{1}{2}\frac{d_0\epsilon(v_\infty{}^2 + L_0 u_\infty{}^2)}{\rho\beta K} - u_\infty\left(1-\frac{a^3}{r^3}\right) r\cos\theta + C\frac{a^2}{r} \quad . \qquad [3.2]$$

By calculating v and w from [2.7] and using [2.20] one obtains

$$w_0 = \epsilon C. \qquad [3.3]$$

Let us introduce parameters

$$\alpha = \frac{\beta K C}{(d_1 - d_0)g}, \qquad \gamma^2 = \frac{w_\infty{}^2}{(1-d_0/d_1)ga}, \qquad s = \frac{u_\infty}{w_\infty} \quad . \qquad [3.4]$$

Equations [2.22] lead then to a requirement

$$2(\cos\theta + \alpha) + \gamma^2(1 - \tfrac{9}{4}\sin^2\theta + (L_1 - \epsilon^2\alpha^2)s^2) = 0, \qquad [3.5]$$

which should be fulfilled over the whole surface $r = a$. Evidently this is not the case because, as is being discussed in section 2 the bubble form has to be determined while solving the problem and cannot be chosen beforehand. Therefore, following a general method by Davies & Taylor (1950) we shall require equation [3.5] to be satisfied only at the critical frontal point on the bubble surface and in its nearest vicinity. By equating the first and second terms of the expansion of [3.5] in degrees of $\sin^2\theta$ at $\theta \approx \pi$ to zero, we get

$$\gamma^2 = \tfrac{4}{9}, \qquad \alpha = \tfrac{7}{9} - \tfrac{2}{9}(L_1 - \epsilon^2\alpha^2)s^2. \qquad [3.6]$$

The velocity of the bubble gravity centre x_c in the system of co-ordinates shown in figure 1 equals zero in the case under study so that w_∞ coincides with the rise velocity U of the bubble, i.e. from [3.4] and [3.6] an equation follows

$$w_\infty = U = \frac{2}{3}\left(\left(1-\frac{d_0}{d_1}\right)ga\right)^{1/2}. \qquad [3.7]$$

This is the well-known Taylor's formula for the velocity of a large bubble or drop rising in an unbounded liquid. For fluidization bubbles the same formula was derived by Davidson & Harrison (1963) and others.

The second equation in [3.6] has two roots, only one of them remaining finite at all s. The latter is

$$\alpha = \frac{9}{4\epsilon^2 s^2}\left\{1 - \left[1 - \frac{8}{9}\epsilon^2 s^2\left(\frac{7}{9} - \frac{2}{9}L_1 s^2\right)\right]^{1/2}\right\}, \qquad [3.8]$$

the constant C being defined in terms of α in accordance with [3.4]. Thus, all the wanted parameters are expressed through known quantities by [3.3], [3.7] and [3.8] and this defines completely the potentials ϕ and ψ in [3.1] and [3.2].

The quantity α characterizes the rate of change of the bubble volume during its motion in the bed. Really, by taking into account [2.15] one derives

$$\frac{dV}{dt} = -\epsilon \oint \frac{\partial \psi}{\partial r} d\Gamma = 4\pi a^2 \alpha \epsilon u_\infty. \tag{3.9}$$

Usually $\epsilon \approx 0.4$–0.5, $L_1 \sim 1$ so that only s defined in [3.4] substantially affects the quantity α. The latter is a monotonously decreasing function of s and changes its sign at $s = s_*$,

$$s_* = (7/2L_1)^{1/2}, \tag{3.10}$$

(see a curve $\alpha = \alpha(s)$ drawn for $\epsilon = 0.4$ and $L_1 = 1$ in figure 2). The corresponding critical value of the bubble radius is

$$a_* = \frac{9L_1}{14} \frac{u_\infty^2}{(1 - d_0/d_1)g} \tag{3.11}$$

([3.4], [3.7] and [3.10] were used).

Hence it follows that a bubble of constant volume is unstable, it is either growing or shrinking when moving in the bed. There exists a critical value of the bubble volume such that larger bubbles grow and smaller ones vanish when rising. If one remembers the meaning of the parameter s from [3.4], one may conclude that the lower the ratio of the interstitial fluidization velocity to the rise velocity, the higher the growth rate parameter α. The effect of changing of the bubble volume is apparently associated with the necessity for a moving cavity to maintain equilibrium at its boundary with a two-phase mixture. It appears that the static equilibrium is impossible and the velocity of the particulate phase at the bubble surface adjusts itself in such a way as to meet conditions required for existence of the state of dynamic equilibrium. A similar point of view was intuitively adopted by Rowe & Matsuno (1971) who attributed this effect, however, to lack of equilibrium between the fluid inside the bubble and the interstitial fluid.

It is possible to observe this effect in practice on condition that s is sufficiently large, that is the bubble is small and the bed particles are rather coarse and require a high fluid flow-rate to fluidize them. Vanishing of small bubbles was indeed reported by Davies & Richardson (1966). Similar results were obtained by Rowe & Matsuno (1971) who produced bubbles artificially, by means of injecting air pulses into fluidized beds. The latter observations confirm that the form and rising velocity of small bubbles do not vary with the flow-rate.

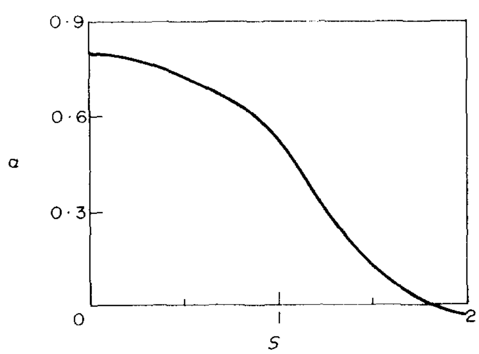


Figure 2. Effect of the velocity ratio s on the parameter α determining the bubble growth rate.

Let us now assume $\xi(\theta)$ to be small as compared with unity but different from zero. Then one can choose some suitable analytical expression for $\xi(\theta)$ depending on a certain number of arbitrary constants. The functions [3.1] and [3.2] are to be regarded as the first approximations to the real potentials. To determine deviations of the latter from the former, one needs to solve a typical problem of regular perturbations. Really, it is easy to derive on the basis of the above formulae two independent problems

$$\Delta\delta\phi = 0, \qquad \delta\phi = 0\,(r \to \infty), \qquad \frac{\partial\delta\phi}{\partial r} = \left(-a\frac{\partial^2\phi}{\partial r^2} + w_0\right)\xi, \qquad (r = a), \tag{3.12}$$

$$\Delta\delta\psi = 0, \qquad \delta\psi = 0\,(r \to \infty), \qquad \delta\psi = -a\xi\frac{\partial\psi}{\partial r}, \qquad (r = a), \tag{3.13}$$

which can be solved in a trivial manner. The solution of [3.12] and [3.13] depends upon the arbitrary constants in the definition of $\xi(\theta)$ which have to be chosen so as to ensure [2.22] to be satisfied not only near the critical point but also at other points on the bubble surface, the number of points being equal to that of the constants. This provides equations for actual calculation of these constants in the same way as equations [3.6] provide for determination of U and $\mathrm{d}V/\mathrm{d}t$. One may hope to obtain an approximate solution differing rather slightly from the exact one by increasing the number of arbitrary parameters in $\xi(\theta)$.

4. LARGE BUBBLES

Consider now a large bubble whose form can be approximately visualized as that of a spherical cap with the semi-vertical angle $\pi - \theta_*$ subtended at the centre $r = 0$ as shown in figure 1. There is a wake region behind the bubble where something like a closed circulation flow of the particulate phase occurs. The surface Γ' enclosing both bubble and wake can be roughly regarded as a sphere whose radius coincides with the curvature radius a of the bubble upper boundary. The same idea is usually adopted for large bubbles rising through extended liquids (Davies & Taylor 1950). The gravity centre of the bubble is placed at the distance x_c above the apex $r = 0$ and

$$x_c = aF(\theta_*), \quad F(\theta_*) = \frac{3}{8}\frac{1 - \cos^2\theta_* + \cos^4\theta_*}{(1 + \cos\theta_*)^2(1 - 0.5\cos\theta_*)}. \tag{4.1}$$

The bubble volume

$$V = \tfrac{4}{3}\pi\kappa(\theta_*)a^3, \qquad \kappa(\theta_*) = \tfrac{1}{2}(1 + \cos\theta_*)^2(1 - \tfrac{1}{2}\cos\theta_*), \tag{4.2}$$

so that $\kappa(\theta_*)$ plays a role of the fraction by volume occupied by the bubble within the sphere $r = a$.

It is evident that the rise velocity U defined as the velocity of the bubble gravity centre in the laboratory co-ordinate system is related to the velocity w_∞ as follows:

$$U = w_\infty + w_c = w_\infty + w_0\frac{x_c}{a} = w_\infty + w_0F(\theta_*), \tag{4.3}$$

where w_c is the gravity centre velocity in the co-ordinate system moving with the origin $r = 0$, $F(\theta_*)$ being defined in [4.1]. An obvious inequality

$$U \sim w_\infty \gg w_0 \sim u_\infty \tag{4.4}$$

holds true for sufficiently large bubbles.

The particulate phase flow is irrotational only outside Γ', a boundary condition on Γ' being unknown. However, this difficulty can be avoided if one obtains, by making use of [4.4], an approximate expression for ϕ valid in the vicinity of Γ'

$$\phi \approx w_\infty\left(1+\frac{a^3}{2r^3}\right)r\cos\theta, \qquad r\sim a. \qquad [4.5]$$

Generally speaking, this equation is derived while neglecting terms of the order of u_∞/U which are due to the unknown normal component of the particulate phase velocity on Γ'. The corresponding error in an expression for $|\nabla\phi|^2$ appearing in [2.22] can still be shown to be of the order of u_∞^2/U^2. On the contrary, far away from the bubble one has

$$\phi \approx w_\infty r\cos\theta - \kappa(\theta_*)w_0\frac{a^2}{r}, \qquad r\gg a, \qquad [4.6]$$

$$\psi \approx -u_\infty r\cos\theta + C\frac{a^2}{r}, \qquad r\gg a, \qquad [4.7]$$

the latter terms in [4.6] and [4.7] describing a symmetric radial flow of both phases due to a change in the bubble volume and $\kappa(\theta_*)$ being expressed in [4.2].

By taking into account [2.20], [4.6] and [4.7] and performing the integration in [2.20] over a sphere of a large radius $r\gg a$, one derives instead of [3.3]

$$w_0 = \frac{\epsilon}{\kappa(\theta_*)}C. \qquad [4.8]$$

It is quite natural to neglect terms of the order of u_∞^2 as compared with those of the order of U^2 when writing [2.22]. Hence an equation

$$2(\cos\theta+\alpha)+\gamma^2(1-\tfrac{9}{4}\sin^2\theta)=0 \qquad [4.9]$$

results, [4.5] being accounted for and the parameters α and γ being determined in [3.4]. This equation replaces [3.5] and its error is of the order of u_∞^2/U^2. By considering, as before, the terms proportional to unity and $\sin^2\theta$ in the expansion of [4.9] one obtains further

$$\gamma^2=\tfrac{4}{9}, \qquad \alpha=\tfrac{7}{9} \qquad [4.10]$$

instead of [3.6]. From [4.3], [4.8] and [4.10] an expression for U follows

$$U=\frac{2}{3}\left(\left(1-\frac{d_0}{d_1}\right)ga\right)^{1/2}+\Phi(\theta_*)\epsilon u_\infty, \qquad \Phi(\theta_*)=\frac{F(\theta_*)}{\kappa(\theta_*)}. \qquad [4.11]$$

According to [4.11] the rise velocity of real, large bubbles in a fluidized bed deviates from that of large bubbles in a liquid of the same density. The former velocity tends, nevertheless, to the latter as the bubble radius grows so that the corresponding ratio u_∞/U decreases. This conclusion agrees well with the analysis of experimental data presented in Murray (1965). It follows from [4.11] that U increases when $\Phi(\theta_*)$ and ϵu_∞ grow, in conformity with some experimental results (see, e.g. Rowe & Partridge 1965).

Note that [4.11] is obtained under the condition $u_\infty/U\ll 1$. However, the second term on its right-hand side can be significant even in this case because Φ is large (see below). If $u_\infty/U\sim 0.1$ and $\epsilon\Phi = 1$–2, the velocity of bubbles in a fluidized bed is about 10–20% higher than that for bubbles in a pure liquid.

The rate of volume change is given by [3.9] with $\alpha = 7/9$. Bearing in mind that $U=\mathrm{d}H/\mathrm{d}t$, H

being the vertical coordinate of the bubble gravity centre in the laboratory system of co-ordinates, one derives from [3.9] and [4.11] an equation

$$\frac{dV}{dH} = \frac{14\pi}{3}\left(\frac{d_1}{d_1 - d_0}\right)^{1/2} \frac{\epsilon u_\infty a^{3/2}}{g^{1/2}} = 7\left(\frac{\pi}{3}\frac{d_1}{d_1 - d_0}\right)^{1/2} \frac{\epsilon u_\infty}{((\kappa(\theta_*)g)^{1/2}} V^{1/2} \tag{4.12}$$

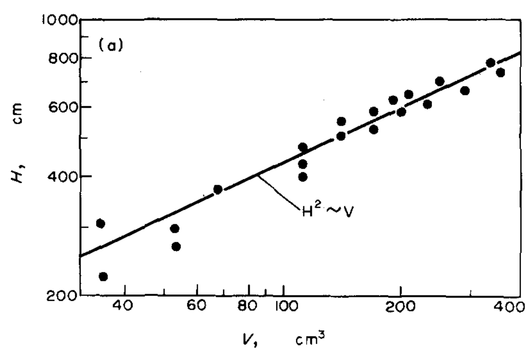
which is accurate to terms of the first order in u_∞/U. This quantity was experimentally investigated by Davies & Richardson (1966) who concluded dV/dH to be a linear function of V. This conclusion contradicts [4.12], but a careful perusal of experimental results in the paper cited shows that experimental points can be correlated with curves $dV/dH \sim V^{1/2}$ with the same success as with curves $dV/dH \sim V$. Examples of such a correlation are shown in figure 3 where co-ordinates V and H are used, theoretical curves $V \sim H^2$ following from [4.12]. Note that results in Davies & Richardson (1966) confirm also the linear dependence [4.12] of the quantity dV/dH upon the superficial velocity ϵu_∞.

In order to determine the potential ψ one needs to solve the Laplace equation in [2.8] under conditions

$$\nabla\psi = \mathbf{u}_\infty(r \gg a), \qquad \psi = C(\mathbf{r} \in \Gamma) \tag{4.13}$$

resulting from [2.13], [2.23] with C expressed in terms of $\alpha = 7/9$ in accordance with [3.4]. Here we do not consider this problem; its solution is apparently needed, however, for investigation of the fluid streamlines.

Equations [4.11] and [4.12] contain an unknown angle θ_* determining a relation between the curvature radius of the bubble upper surface and the bubble volume. To find it we use a rather simple reasoning based on the mental consideration of a possible iteration procedure aimed to determination of the potential ϕ of the particulate phase flow in a case when the surface Γ' differs slightly from a spherical one. Such a procedure can be constructed in the following manner.



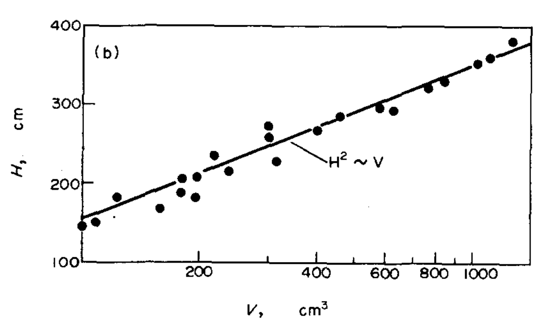


Figure 3. Comparison of theoretical dependences $H^2 \sim V$ with experimental data in Davies & Richardson (1966) on air-fludized beds of irregular craker catalyst particles of mean diameter 0·0055 cm. (a) $\epsilon u_\infty = 0.252$ cm/sec; (b) $\epsilon u_\infty = 0.332$ cm/sec.

Let the sphere $r = a$ be the zeroth approximation to the real surface Γ' which may be now defined with the help of [2.12]. Then $\xi^{(0)}(\theta) = 0$ and the potential is given by [4.5]. At the surface [2.12] with $\xi(\theta) = \xi^{(1)}(\theta)$ we have

$$\phi^{(0)} = \tfrac{3}{2} w_\infty a(1 - \xi^{(1)}(\theta)) \cos \theta, \qquad [4.14]$$

$$|\nabla \phi^{(0)}|^2 = \tfrac{9}{4} w_\infty^2 (1 - 2\xi^{(1)}(\theta)) \sin^2 \theta, \qquad [4.15]$$

superscripts designating the interaction numbers. Introducing [4.14] and [4.15] into [2.22] enables us to find $\xi^{(1)}(\theta)$ corresponding to the common part $\theta > \theta_*$ of Γ and Γ' as a solution of this equation. Let us imagine further $\xi^{(1)}(\theta)$ to be also extrapolated to smaller values of θ in a certain suitable way which is of no special interest here. Considering further a problem for the potential $\phi^{(1)}$ outside the surface Γ' defined with the help of $\xi^{(1)}(\theta)$ we are in principle able to obtain an expression for $\phi^{(1)}$, to use in [2.22] again and to calculate the second approximation $\xi^{(2)}(\theta)$, and so forth. This procedure is evidently regular and self-consistent and resulting successive problems resemble the problem [3.12], if $\xi^{(1)}(\theta)$ and other iterations are actually small compared with unity.

Having in mind to evaluate the critical angle θ_*, it is fairly natural to identify it with just an angle when the condition of smallness of $\xi^{(1)}(\theta)$ ceases to hold. By making use of [4.14] and [4.15] instead of [4.5] one gets the following equation

$$2[\alpha + (1 + \xi^{(1)}) \cos \theta] + \gamma^2 [1 - \tfrac{9}{4}(1 - 2\xi^{(1)}) \sin^2 \theta] = 0, \qquad [4.16]$$

with α and γ being already expressed in [4.10]. This equation replaces [4.9] and reflects the influence upon the momentum balance at the bubble surface of slight deviations of this surface from a sphere. According to the iteration procedure suggested above, it must serve for the determination of the function $\xi^{(1)}(\theta)$. By resolving [4.16] and accounting for [4.10] one obtains

$$\xi^{(1)}(\theta) = -\frac{2(\tfrac{7}{9} + \cos \theta) + \tfrac{4}{9}(1 - \tfrac{9}{4} \sin^2 \theta)}{2(\cos \theta + \sin^2 \theta)}. \qquad [4.17]$$

The function [4.17] tends to infinity as θ comes to θ_* which is a root of an equation

$$\cos \theta_* + \sin^2 \theta_* = 0. \qquad [4.18]$$

Hence

$$\cos \theta_* = -0.62, \qquad \pi - \theta_* = 51°40'. \qquad [4.19]$$

The corresponding values of various functions of θ_* involved in above formulae are

$$\kappa = 0.095, \qquad F = 0.75, \qquad \Phi = 7.9. \qquad [4.20]$$

This brings the analysis to completion.

Let us emphasize that the same reasoning is also applicable to a large bubble or drop moving in an extended liquid. In this case one obtains in a similar manner an equation

$$2[1 + (1 + \xi) \cos \theta] - \tfrac{9}{4} \gamma^2 (1 - 2\xi) \sin^2 \theta = 0 \qquad [4.21]$$

replacing [4.16] and other equations of exactly the same form as [4.18]–[4.20]. That is why [4.19] permits solution of the famous problem in Davies & Taylor (1950) in that not only a relation between the rise velocity and the curvature radius is known but also [4.1], enabling this radius to be found in terms of the bubble volume. There is a significant scatter of experimental values of $\pi - \theta_*$ varying for large bubbles from 46° to 64° (Batchelor 1970). A thorough analysis of very large bubbles, when surface tension effects are surely of no importance, gave $\pi - \theta_* \approx 50°$ (see, e.g. Davenport *et al.* 1967).

Surprisingly enough, this value of the semi-vertical angle is in a good agreement with the result [4.19], the latter being merely a qualitative one. This agreement encourages one to proceed to the investigation of the surface tension influence upon the bubble form. It is not difficult to derive an equation

$$1+(1+\xi)\cos\theta-\tfrac{9}{8}\gamma^2(1-2\xi)\sin^2\theta-W\left[2\xi+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\xi}{\partial\theta}\right)\right]=0, \qquad [4.22]$$

which reduces to [4.21] when a parameter

$$W=\frac{\sigma}{(d_1-d_0)ga^2} \qquad [4.23]$$

goes to zero, σ being the surface tension coefficient. Here d_0 and d_1 are understood as the densities of the inner and external fluids and γ is defined by [3.4], w_∞ playing in this case a role of the bubble rise velocity. Equation [4.22] is a differential equation of the second order which can be solved numerically at any W. Unfortunately, its solutions $\xi(\theta, W)$ increase with $\pi-\theta$ in a smooth manner, without tending to infinity when θ turns to some finite value $\theta_*(W)$, and there is no critical value of θ which could be unambiguously associated with θ_*. Nevertheless, when confining ourselves with a qualitative analysis, it is certainly of use to study solutions of [4.22] at different W. The dependence of a value of $\pi-\theta_*$ defined by a requirement $\xi(\theta_*)\approx 10$ upon the parameter W from [4.23] is shown in figure 4. It can be clearly seen that a rather small W induces a drastic change to the angle $\pi-\theta_*$ as compared with $\pi-\theta_*$ at $W=0$. Thus, no matter how small parameter W, the scatter of observed values of θ_* can be presumably attributed to the action of surface tension forces.

Turning again to discussion of fluidization bubbles encountered in practice one concludes them to be somewhat "thicker" than the bubble corresponding to [4.19], that is real values of $\pi-\theta_*$ are larger than 50°. This seems to be due to the action of an effective "surface tension", being displayed on boundaries between a fluid–solid mixture and the pure fluid, in the same way as is the case for bubbles in liquids. A detailed study of processes occurring at such boundaries was carried out in Buyevich & Gupalo (1970) where a physical origin of forces resembling those of true surface tension and a model allowing these forces to be evaluated were put forward. To judge by results obtained there, the effective surface tension coefficient must vary along the bubble surface and its mean value representing a monotonously increasing function of u_∞ can amount to several dynes per centimetre.†

In conclusion, we point out one important consequence which is usually ignored. Although the bubble growth caused by the regular inflow of the fluid phase may be insignificant in many cases as compared with the growth of real bubbles due to coalescence, the former is of primary importance while treating mass and heat exchange processes between bubbles and the "continuous" phase of a fluidized bed. Really, the radial motion of the fluid flowing into a bubble makes conditions for approach of some dispersed substance to the bubble far more favourable. On the other hand, the same flow hinders to an essential extent the transfer in the opposite direction, i.e. from bubbles to

†In this connection the author wishes to stress that the above remarks on possible influence of surface tension effects upon the form of bubbles in both one-phase liquids and fluidized beds have to be understood as purely suggestive or thought-leading ones. There are too many complex physical factors involved and each of them needs further clacification so that there remains much work to be done in order to elucidate this point quite unambiguously. Nevertheless, it is the private author's opinion that the effective surface tension has more to do with the observed bubble form (at any rate, for bubbles within a fluidized bed) than, for example, viscosity effects which are hardly of especial importance because of mobility of the free bubble surface.

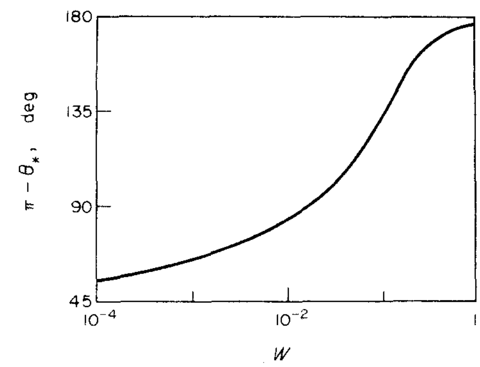


Figure 4. The dependence of the semi-vertical angle defining the form of large bubbles in liquids upon the Weber number (see explanations in the text).

the continuous phase. Keeping in mind that the specific heat capacity of bed particles exceeds considerably, as a rule, that of the fluid phase, we are able to conclude the radial particle motion induced by growing bubbles to have even more substantial influence on heat transfer between bubbles and the continuous phase.

These effects are not taken into account in the majority of papers on this subject (see, for example, recent papers by Drinkenburg & Rietema 1972, 1973). However, a preliminary order-of-magnitude consideration evidences that the radial flow of both phases exerts under certain conditions a decisive influence on heat and mass exchange processes in fluidized beds and must be incorporated in any modern analysis of these processes.

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Résumé—On présente un modèle pour une seule bulle bien développée, se déplaçant dans un lit fluidisé infini. Le modèle prends en compte la croissance ou la décroissance de la bulle pendant son ascension au sien du lit et permet d'expliquer l'influence des paramètres du lit sur la vitesse de montée de la bulle. Les cas-limites de bulles presque sphériques et de bulles suffisamment grosses dont la forme ressemble à celle d'un segment de sphère sont examinés de façon plus détaillée. On discute de la forme des bulles montant soit dans un lit fluidisé soit dans un liquide monophasique et de la façon dont elle dépend de la "tension superficielle" effective à la frontière de la bulle.

Auszug—Fuer eine einzelne, sich in einem unbegrenzten Fliessbett bewegende Blase wird win Modell entwickelt. Es gestattet, das Wachsen oder Einschrumpfen der Blase waehrend ihres Aufsteigens im Bett, wie auch die Abhaengigkeit der Aufstiegsgeschwindigkeit von den Fliessbettparametern zu erklaeren. Die Grenzfaelle fast kugelfoermiger Blasen, und von Blasen ausreichender Groesse und von kugelsegmentaehnlicher Form, werden eingehender betrachtet. Die Form von Blasen, die entweder in Fliessbetten oder in einphasigen Fluessigkeiten aufsteigen, und ihre Abhaengigkeit von einer effektiven, in der Grenzschicht wirkenden "Oberflaechenspannung" wird besprochen.

Резюме—Предложена модель единичного развитого пузыря, движущегося в неограниченном псевдоожиженном слое. Модель позволяет объяснить рост или уменьшение объема пузыря по мере подъема в слое, а также зависимость скорости подъема от параметров слоя. Подробнее рассмотрены предельные случаи малых приблизительно сферических пузырей и больших пузырей, форма которых напоминает сферический сегмент. Обсуждена форма крупных пузырей, поднимающихся как в псевдоожиженном слое, так и в однофазной жидкости, подчеркивается ее зависимость от эффективного «поверхностного натяжения», действующего на границе пузыря.